SAMPLE QUESTION PAPER

Issued by CBSE for 2019 Examinations
Class XII - Mathematics

Time Allowed : 180 Minutes **Max. Marks : 100**

General Instructions :

(a) All questions are compulsory.
(b) This question paper consists of **29 questions** divided into **four sections A, B, C and D**.
(c) Section A comprises of **04 questions of one mark** each (from Q01 – 04).
Section B comprises of **08 questions of two marks** each (from Q05 – 12).
Section C comprises of **11 questions of four marks** each (from Q13 – 23).
Section D comprises of **06 questions of six marks** each (from Q24 – 29).
(d) There is no overall choice. However, **internal choice** has been provided in **01 Question of Section A, 03 Questions of Section B, 03 Questions of Section C** and **03 Questions of Section D, each**. You have to attempt only one of the alternatives in all such questions.

SECTION A

Q01. If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -1/6$, find $|B|$.

Q02. Differentiate $\sin^2 (x^2)$ w. r. t. x^2.

Q03. Write the order of the differential equation :

$$\log\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^3 + x.$$

Q04. Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, n$ makes with positive direction of z-axis.

OR Find the direction cosines of the line $\frac{x-1}{2} = -y = \frac{z+1}{2}$.

SECTION B

Q05. Let $A = Z \times Z$ and * be a binary operation on A defined by (a, b) * (c, d) = (ad + bc, bd). Find the identity element for * in the set A.

Q06. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + kI$.

Q07. Find $\int \frac{(x^2 + \sin^2 x)\sec^2 x}{1+x^2} dx$.

Q08. Find $\int \frac{e^x (x-3)}{(x-1)^3} dx$.

OR Find $\int \frac{(x^4 - x)^{1/4}}{x^5} dx$.

Q09. Form the differential equation of all circles which touch the x-axis at the origin.

Q10. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

OR Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

Q11. If A and B are two independent events, prove that A' and B are also independent.

Q12. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

OR If P(A) = 0.6, P(B) = 0.5 and P(A|B) = 0.3, then find $P(A \cup B)$.

SECTION C

Q13. Prove that the function $f:[0,\infty)\to R$ given by $f(x)=9x^2+6x-5$ is not invertible. Modify the codomain of the function f to make it invertible, and hence find f^{-1}.

OR Check whether the relation R in the set **R** of real numbers, defined by $R=\{(a,b):1+ab>0\}$, is reflexive, symmetric or transitive.

Q14. Find the value of $\sin\left(2\tan^{-1}\frac{1}{4}\right)+\cos(\tan^{-1}2\sqrt{2})$.

Q15. Using the properties of determinants, prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}=(a+b+c)(a^2+b^2+c^2)$.

Q16. If $y=x^{\sin x}+\sin(x^x)$, find $\frac{dy}{dx}$.

OR If $y=\log(1+2t^2+t^4),\ x=\tan^{-1}t$, find $\frac{d^2y}{dx^2}$.

Q17. If $y=\cos(m\cos^{-1}x)$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

Q18. Find the equations of the normal to the curve $y=4x^3-3x+5$ which are perpendicular to the line $9x-y+5=0$.

Q19. Find $\int\frac{x^4+1}{x(x^2+1)^2}dx$.

Q20. Evaluate $\int_{-1}^{1}\frac{x+|x|+1}{x^2+2|x|+1}dx$.

Q21. Find the particular solution of the following differential equation :

$$\cos y\,dx+(1+2e^{-x})\sin y\,dy=0;\ y(0)=\frac{\pi}{4}.$$

OR Find the general solution of the differential equation :

$$\frac{dx}{dy}=\frac{y\tan y-x\tan y-xy}{y\tan y}.$$

Q22. If $\vec{p}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector $\vec{q}$ and coplanar with vectors $\vec{p}$ and $\vec{q}$.

Q23. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

SECTION D

Q24. If $A=\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}.

Hence, solve the system of equations : $3x+3y+2z=1,\ x+2y=4,\ 2x-3y-z=5$.

OR Find the inverse of the following matrix using elementary transformations :

$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}.$$

Q25. A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square meter is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and walls.

Q26. Find the area bounded by the curves $y=\sqrt{x}$, $2y+3=x$ and x-axis.

OR Find the area of the region $\{(x,y): x^2+y^2 \le 8,\ x^2 \le 2y\}$.

Q27. Find the equation of the plane containing the line $\frac{x-1}{3}=\frac{y-4}{2}=\frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2}=\frac{1-y}{4}=\frac{z+2}{1}$. Hence, find the shortest distance between the lines.

OR Show that the line of intersection of the planes $x+2y+3z=8$ and $2x+3y+4z=11$ is coplanar with the line $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$. Also find the equation of the plane containing them.

Q28. A manufacturer makes two types of toys A and B. Three machine are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Type of Toys	**Machines**		
	I	II	III
A	20	10	10
B	10	20	30

The machines I, II and III are available for a maximum of 3 hours, 2 hours and 2 hours 30 minutes respectively. The profit on each toy of type A is ₹50 and that of type B is ₹60. Formulate the above problem as a L. P. P. and solve it graphically to maximize profit.

Q29. The members of a consulting firm rent cars from three rental agencies :
50% from agency X, 30% from agency Y and 20% from agency Z.
From past experience, it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting. If the rental car delivered to the firm need service and tuning, find the probability that agency Z is not to be blamed.

MARKING SCHEME

Q01. As $\left|P^{-1}\right|=\frac{1}{|P|}$ $\therefore\left|(AB)^{-1}\right|=\frac{1}{|AB|}=\frac{1}{|A||B|}=\frac{1}{2\times|B|}$

Also since $\left|(AB)^{-1}\right|=-\frac{1}{6}$ so, $\frac{1}{2\times|B|}=-\frac{1}{6}$ $\therefore|B|=-3$. [1]

Q02. Let $y=\sin^2(x^2)$ and, $z=x^2$ $\therefore\frac{dy}{dx}=2\sin(x^2)\cos(x^2)\times2x=2x\sin(2x^2)$ and $\frac{dz}{dx}=2x$

Now $\frac{dy}{dz}=\frac{dy}{dx}\times\frac{dx}{dz}=2x\sin(2x^2)\times\frac{1}{2x}=\sin(2x^2)$ or, $2\sin(x^2)\cos(x^2)$. [1]

Alternative : $\frac{d}{dx^2}\left(\sin^2(x^2)\right)=2\sin(x^2)\times\frac{d}{dx^2}\left(\sin(x^2)\right)=2\sin(x^2)\times\cos(x^2)\frac{d}{dx^2}\left(x^2\right)$

$=2\sin(x^2)\cos(x^2)\times1=\sin(2x^2)$.

Q03. Order of the differential equation $\log\left(\frac{d^2y}{dx^2}\right)=\left(\frac{dy}{dx}\right)^3+x$ is : 2. [1]

Q04. As $l^2+m^2+n^2=1$ $\therefore\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{6}}\right)^2+n^2=1$ $\Rightarrow n=\frac{1}{\sqrt{2}}=\cos\gamma$ $\therefore\gamma=\frac{\pi}{4}$. [1]

Here γ is the **acute** angle made by the line with positive direction of z-axis.

OR On Rewriting the line in Symmetrical form : $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+1}{2}$.

So, the d.r's of the line are 2, –1, 2. [1/2]

Hence the d.c.'s of the line are $\pm\frac{2}{\sqrt{2^2+(-1)^2+2^2}},\pm\frac{-1}{\sqrt{4+1+4}},\pm\frac{2}{3}$ i.e., $\pm\frac{2}{3},\mp\frac{1}{3},\pm\frac{2}{3}$. [1/2]

SECTION B

Q05. Let $(e,f)\in A=Z\times Z$ be the identity element for *.

So, $(a,b)*(e,f)=(a,b)=(e,f)*(a,b)\ \forall\ (a,b)\in A$ [1/2]

i.e., if $(af+be,bf)=(a,b)=(eb+fa,fb)$

Consider $(af+be,bf)=(a,b)$ and $(a,b)=(eb+fa,fb)$

i.e., $af+be=a, bf=b$ and $a=eb+fa, b=fb$

i.e., $a\times1+be=a, f=1$ and $a=eb+1\times a, 1=f$ (if $b\in Z-0$, i.e., $b\neq0$)

i.e., $e=0, f=1$ and $0=e, 1=f$. [1]

Hence (0, 1) is the identity element. [1/2]

Q06. $A^2=\begin{bmatrix}3&1\\-1&2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}=\begin{bmatrix}8&5\\-5&3\end{bmatrix}, 5A=\begin{bmatrix}15&5\\-5&10\end{bmatrix}, kI=\begin{bmatrix}k&0\\0&k\end{bmatrix}$ [1/2 + 1/2]

Now $A^2=5A+kI$ so, $\begin{bmatrix}8&5\\-5&3\end{bmatrix}=\begin{bmatrix}15&5\\-5&10\end{bmatrix}+\begin{bmatrix}k&0\\0&k\end{bmatrix}$ $\Rightarrow\begin{bmatrix}8&5\\-5&3\end{bmatrix}=\begin{bmatrix}15+k&5\\-5&10+k\end{bmatrix}$

On comparing the corresponding elements in both matrices, we get :

$8=15+k, 3=10+k$ $\therefore k=-7$. [1]

Q07. Let $I=\int\frac{(x^2+\sin^2x)\sec^2x}{1+x^2}dx=\int\frac{(1+x^2+\sin^2x-1)\sec^2x}{1+x^2}dx$ [1/2]

$\Rightarrow I=\int\frac{(1+x^2-\cos^2x)\sec^2x}{1+x^2}dx=\int\frac{(1+x^2)\sec^2x-1}{1+x^2}dx$ [1/2]

$\Rightarrow I=\int\left[\sec^2x-\frac{1}{1+x^2}\right]dx=\tan x-\tan^{-1}x+C$. [1]

Q08. Let $I=\int \frac{e^x(x-3)}{(x-1)^3}dx=\int e^x\left[\frac{(x-1)-2}{(x-1)^3}\right]dx=\int e^x\left[\frac{1}{(x-1)^2}+\frac{-2}{(x-1)^3}\right]dx$ [1/2 + 1/2]

$\therefore I=e^x\left[\frac{1}{(x-1)^2}\right]+C$ $\qquad [\because \int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+C.$ [1]

OR Let $I=\int \frac{(x^4-x)^{1/4}}{x^5}dx=\int\left\{x^4\left(1-\frac{x}{x^4}\right)\right\}^{1/4}\frac{1}{x^5}dx=\int x\left(1-\frac{1}{x^3}\right)^{1/4}\frac{1}{x^5}dx$ [1/2]

$\Rightarrow I=\int\left(1-\frac{1}{x^3}\right)^{1/4}\frac{1}{x^4}dx$ $\qquad \left[\text{Put } 1-\frac{1}{x^3}=t \Rightarrow \frac{dx}{x^4}=\frac{dt}{3}\right.$ [1/2]

$\therefore I=\frac{1}{3}\int t^{1/4}dt=\frac{1}{3}\times\frac{4}{5}t^{5/4}+C=\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4}+C.$ [1]

Q09. Let r be the radius of all circles which touch the x-axis at origin. So, centre of all such circles must lie on y-axis. Therefore, the centre will be of the form (0, r).
So, the equation of all such circles : $(x-0)^2+(y-r)^2=r^2$ i.e., $x^2+y^2-2ry=0$

Rearranging the terms, we get : $\frac{x^2}{y}+y=2r$ [1]

Now differentiating w.r.t. x : $\frac{y(2x)-x^2y'}{y^2}+y'=0$ i.e., $(x^2-y^2)y'=2xy$. [1]

Q10. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 4\\ 2 & -1 & 2\end{vmatrix}=-2\hat{i}+4\hat{j}+4\hat{k}$ [1]

Now, area of the parallelogram $=\frac{\left|\vec{a}\times\vec{b}\right|}{2}=\frac{\sqrt{4+16+16}}{2}=\frac{6}{2}=3$ sq. units . [1/2 + 1/2]

OR Let θ be the required angle between given vectors.

So, $\cos\theta=\frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}=\frac{(\hat{i}+\hat{j}-\hat{k})(\hat{i}-\hat{j}+\hat{k})}{\sqrt{1+1+1}\sqrt{1+1+1}}=\frac{1-1-1}{\sqrt{3}\sqrt{3}}=-\frac{1}{3}$ $\qquad \therefore \theta=\cos^{-1}\left(-\frac{1}{3}\right).$ [1/2 + 1 + 1/2]

Q11. $P(A'\cap B)=P(B-A)=P(B)-P(A\cap B)=P(B)-P(A)P(B)$ [1/2]

$[\because P(A\cap B)=P(A)P(B)$, if A and B are independent events. [1/2]

$\Rightarrow P(A'\cap B)=P(B)[1-P(A)]$ $\qquad \therefore P(A'\cap B)=P(A')P(B)$ [1/2]

Hence, A' and B are also independent if A and B are independent events. [1/2]

Q12. P(Red transferred and red drawn **or** black transferred and red drawn) $=\frac{3}{8}\times\frac{7}{11}+\frac{5}{8}\times\frac{6}{11}=\frac{51}{88}$.
[1 + 1]

Alternative : Let E : the ball drawn from second bag is red, E_1 : red ball is transferred from first bag and E_2 : black ball is transferred from first bag.

$\therefore P(E_1)=3/8,\ P(E_2)=5/8,\ P(E\,|\,E_1)=7/11,\ P(E\,|\,E_2)=6/11$

So, $P(E)=P(E_1)P(E\,|\,E_1)+P(E_2)P(E\,|\,E_2)=\frac{3}{8}\times\frac{7}{11}+\frac{5}{8}\times\frac{6}{11}=\frac{51}{88}$.

Note that this problem is based on Total Probability.

OR We have $P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)+P(B)-P(A\cup B)}{P(B)}$ [1/2]

$\Rightarrow 0.3=\frac{0.6+0.5-P(A\cup B)}{0.5}$ $\qquad \Rightarrow 0.15-1.1=-P(A\cup B)$ $\qquad \therefore P(A\cup B)=0.95$. [1½]

SECTION C

Q13. First of all, let's check the function $f(x)$ for one-one.

Let $x_1, x_2 \in [0,\infty)$ such that $f(x_1) = f(x_2)$.

i.e., $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$ $\Rightarrow 9(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)\{9(x_1 + x_2) + 6\} = 0$ $\Rightarrow (x_1 - x_2) = 0$ as $9(x_1 + x_2) + 6 \neq 0$ for $x_1, x_2 \in [0,\infty)$

$\therefore x_1 = x_2$. So, $f(x)$ is one-one. [1]

Now we'll check the function $f(x)$ for onto.

Let $y \in R$, so for any value of $x \in [0,\infty)$, $y = f(x) = 9x^2 + 6x - 5$

i.e., $y = (3x)^2 + 2(3x).1 + 1^2 - 1^2 - 5 = (3x+1)^2 - 6$

i.e., $(3x+1)^2 = y + 6$ $\Rightarrow x = \dfrac{\pm\sqrt{y+6} - 1}{3}$

$\therefore x = \dfrac{\sqrt{y+6} - 1}{3}$ [As $x = \dfrac{-\sqrt{y+6} - 1}{3} \notin [0,\infty)$

Now for $y = -6 \in R$, $x = -\dfrac{1}{3} \notin [0,\infty)$.

Hence $f(x)$ is not onto so, $f(x)$ is not invertible. [1]

Now note that, we have $x \in [0,\infty)$ i.e., $x \geq 0$ so, $\dfrac{\sqrt{y+6} - 1}{3} \geq 0$

$\Rightarrow \sqrt{y+6} \geq 1$ $\Rightarrow y + 6 \geq 1$ $\Rightarrow y \geq -5$. [1]

So, if f is redefined as $f : [0,\infty) \to [-5,\infty)$ then $f(x) = 9x^2 + 6x - 5$ becomes an onto function.

Thus, $f(x)$ is one-one and onto both if $f : [0,\infty) \to [-5,\infty)$. Hence, f is now invertible.

And, $f^{-1} : [-5,\infty) \to [0,\infty)$ is given by $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$. [1]

OR We have $R = \{(a,b) : 1 + ab > 0\}$, where $a, b \in \mathbf{R}$.

Reflexivity : Since $1 + aa = 1 + a^2 > 0 \ \forall \ a \in \mathbf{R}$. So, (a, a) $\in$ R. Hence, R is reflexive. [1]

Symmetry : Let $a, b \in \mathbf{R}$ and (a, b) $\in$ R.

Clearly $1 + ab > 0$ implies, $1 + ba > 0$ which further implies $(b,a) \in R$.

Hence R is symmetric. [1]

Transitivity : Let $a, b, c \in \mathbf{R}$ and (a, b), (b, c) $\in$ R.

Let $a = -8, b = -1, c = \dfrac{1}{2}$.

As $1 + ab = 1 + (-8)(-1) = 9 > 0$ $\therefore (a,b) \in R$ and, $1 + bc = 1 + (-1)\left(\dfrac{1}{2}\right) = \dfrac{1}{2} > 0$ $\therefore (b,c) \in R$.

But, $1 + ac = 1 + (-8)\left(\dfrac{1}{2}\right) = -3 > 0$, which is false. $\therefore (a,c) \notin R$.

Hence R isn't transitive. [2]

Q14. Let $y = \sin\left(2\tan^{-1}\dfrac{1}{4}\right) + \cos(\tan^{-1} 2\sqrt{2}) = u + v$, where $u = \sin 2\tan^{-1}\dfrac{1}{4}$ and $v = \cos\tan^{-1} 2\sqrt{2}$

$$\therefore u = \sin\left(2\tan^{-1}\frac{1}{4}\right) = \sin\sin^{-1}\left(\frac{2.\frac{1}{4}}{1+\left(\frac{1}{4}\right)^2}\right) = \frac{\frac{2}{4}}{\frac{17}{16}} = \frac{8}{17}$$ [1½]

Also, $v = \cos\tan^{-1} 2\sqrt{2}$ [Put $\tan^{-1} 2\sqrt{2} = \theta \Rightarrow \tan\theta = 2\sqrt{2}$ $\Rightarrow \cos\theta = \dfrac{1}{3}$ $\therefore \theta = \cos^{-1}\dfrac{1}{3}$

$\therefore v = \cos\left(\cos^{-1}\frac{1}{3}\right) = \frac{1}{3}$ [1½]

Now, $y = u + v = \frac{8}{17} + \frac{1}{3} = \frac{41}{51}$. [1]

Note we've used $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in $u = \sin 2\tan^{-1}\frac{1}{4}$.

Q15. LHS : Let $\Delta = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$ By $C_1 \to aC_1$,

$\Rightarrow \Delta = \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$ By $C_1 \to C_1 + bC_2 + cC_3$, [1]

$\Rightarrow \Delta = \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$ Taking $(a^2 + b^2 + c^2)$ common from C_1,

[1]

$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$ By $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, [1/2]

$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & c+a & -b \end{vmatrix}$ Expanding along C_1,

$\Rightarrow \Delta = \frac{a^2+b^2+c^2}{a}(-bc + a^2 + ac + ba + bc)$ [1/2]

$\therefore \Delta = (a^2 + b^2 + c^2)(a + b + c) = \text{RHS}$. [1]

Q16. Let $y = u + v$, where $u = x^{\sin x}$ and $v = \sin(x^x)$ $\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(i)

Now $u = x^{\sin x}$ $\Rightarrow \log u = \log x^{\sin x}$ $\Rightarrow \log u = \sin x \log x$

$\Rightarrow \frac{1}{u}\times\frac{du}{dx} = \sin x \times \frac{1}{x} + \log x \cos x$ $\therefore \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$ [1½]

And, $v = \sin(x^x)$ $\Rightarrow \frac{dv}{dx} = \cos(x^x)\times\frac{d}{dx}(x^x) = \cos(x^x)\times\frac{d}{dx}(e^{\log x^x}) = \cos(x^x)\times\frac{d}{dx}(e^{x\log x})$

$\Rightarrow \frac{dv}{dx} = \cos(x^x)\times\left\{e^{x\log x}\left(x\times\frac{1}{x} + \log x\ .\ 1\right)\right\}$ $\therefore \frac{dv}{dx} = \cos(x^x)\times\left\{x^x\left(1 + \log x\right)\right\}$ [1½]

By (i), $\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right) + x^x(1 + \log x)\cos(x^x)$. [1]

OR We've $y = \log(1 + 2t^2 + t^4) = \log(1+t^2)^2 = 2\log(1+t^2)$ $\Rightarrow \frac{dy}{dt} = 2\times\frac{2t}{1+t^2} = \frac{4t}{1+t^2}$ [1½]

Also $x = \tan^{-1}t$ $\Rightarrow \frac{dx}{dt} = \frac{1}{1+t^2}$ $\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{4t}{1+t^2}}{\frac{1}{1+t^2}} = \frac{4t}{1+t^2}\times\frac{1+t^2}{1} = 4t$ [1/2 + 1]

Now differentiating w.r.t. x both sides, we get : $\frac{d^2y}{dx^2}=4\frac{dt}{dx}$ $\therefore \frac{d^2y}{dx^2}=4(1+t^2)$. [1]

Q17. We've $y=\cos(m\cos^{-1}x)$ $\Rightarrow \frac{dy}{dx}=-\sin(m\cos^{-1}x)\times m\times\left(-\frac{1}{\sqrt{1-x^2}}\right)$ [1]

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx}=m\sin(m\cos^{-1}x)$

On squaring both sides, we get : $(1-x^2)\left(\frac{dy}{dx}\right)^2=m^2\sin^2(m\cos^{-1}x)=m^2\{1-\cos^2(m\cos^{-1}x)\}$

$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2=m^2\{1-y^2\}$ [1½]

On differentiating again w.r.t. x : $(1-x^2)2\left(\frac{dy}{dx}\right)\times\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=m^2\left(-2y\frac{dy}{dx}\right)$ [1]

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=m^2(-y)$ $\therefore (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$. [1/2]

Alternative : We've $y=\cos(m\cos^{-1}x)$ $\Rightarrow \frac{dy}{dx}=-\sin(m\cos^{-1}x)\times m\times\left(-\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx}=m\sin(m\cos^{-1}x)$

On differentiating again w.r.t. x : $\sqrt{1-x^2}\frac{d^2y}{dx^2}-\frac{2x}{2\sqrt{1-x^2}}\times\frac{dy}{dx}=m\cos(m\cos^{-1}x)\times\frac{-m}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-m^2y$ $\therefore (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

Q18. Let the normal to the curve $y=4x^3-3x+5$ be at $P(\alpha,\beta)$. $\therefore \beta=4\alpha^3-3\alpha+5$...(i)

Now $\frac{dy}{dx}=12x^2-3$ $\therefore m_N=-\frac{1}{12\alpha^2-3}$ = Slope of normal at $P(\alpha,\beta)$ [1]

As slope of the given line $9x-y+5=0$ is 9 and this line is perpendicular to the normal so,

$\left(-\frac{1}{12\alpha^2-3}\right)\times 9=-1$ i.e., $12\alpha^2-3=9$ $\Rightarrow \alpha^2=1$ $\therefore \alpha=\pm1,\ \beta=6,\ 4$ {By (i)

Therefore the points are (1, 6), (–1, 4). [1]

Equations of Normals are : $y-6=-\frac{1}{9}(x-1)$ i.e., $x+9y=55$ and, [1]

$y-4=-\frac{1}{9}(x+1)$ i.e., $x+9y=35$. [1]

Q19. Let $I=\int\frac{x^4+1}{x(x^2+1)^2}dx=\int\frac{x^4+1+2x^2-2x^2}{x(x^2+1)^2}dx=\int\frac{(x^2+1)^2-2x^2}{x(x^2+1)^2}dx$

$\Rightarrow I=\int\frac{(x^2+1)^2}{x(x^2+1)^2}dx-\int\frac{2x^2}{x(x^2+1)^2}dx$ $\Rightarrow I=\int\frac{1}{x}dx-\int\frac{2x}{(x^2+1)^2}dx$

In second integral, put $x^2+1=y\Rightarrow 2xdx=dy$ $\therefore I=\log|x|-\int\frac{dy}{y^2}=\log|x|+\frac{1}{y}+C$

Thus, $I=\log|x|+\frac{1}{x^2+1}+C$.

Alternative : $I=\int\frac{x^4+1}{x(x^2+1)^2}dx=\int\frac{(x^4+1)x}{x^2(x^2+1)^2}dx$ [Put $x^2=t\Rightarrow xdx=dt/2$ [1/2]

$\Rightarrow I=\frac{1}{2}\int\frac{t^2+1}{t(t+1)^2}dt$ [1/2]

Consider $\frac{t^2+1}{t(t+1)^2}=\frac{A}{t}+\frac{B}{t+1}+\frac{C}{(t+1)^2}$ $\Rightarrow t^2+1=A(t+1)^2+Bt(t+1)+Ct$

On comparing the coefficients of like terms both sides, we get : $A=1, B=0, C=-2$ [1½]

$\therefore I=\frac{1}{2}\int\left\{\frac{1}{t}+\frac{0}{t+1}-\frac{2}{(t+1)^2}\right\}dt=\frac{1}{2}\left\{\log|t|+\frac{2}{t+1}\right\}+C=\frac{1}{2}\left\{\log\left|x^2\right|+\frac{2}{x^2+1}\right\}+C$ [1]

Therefore, $I=\log|x|+\frac{1}{x^2+1}+C$. [1/2]

Q20. Let $I=\int_{-1}^{1}\frac{x+|x|+1}{x^2+2|x|+1}dx=\int_{-1}^{1}\frac{x}{x^2+2|x|+1}dx+\int_{-1}^{1}\frac{|x|+1}{x^2+2|x|+1}dx$ [1]

Let $f(x)=\frac{x}{x^2+2|x|+1}$, $g(x)=\frac{|x|+1}{x^2+2|x|+1}$

$\Rightarrow f(-x)=\frac{-x}{(-x)^2+2|-x|+1}$, $g(-x)=\frac{|-x|+1}{(-x)^2+2|-x|+1}$

$\Rightarrow f(-x)=\frac{-x}{x^2+2|x|+1}$, $g(-x)=\frac{|x|+1}{x^2+2|x|+1}$

$\Rightarrow f(-x)=-f(x),\ g(-x)=g(x)$ $\therefore$ f(x) is odd and g(x) is even function. [1½]

Therefore, $I=0+2\int_0^1\frac{|x|+1}{x^2+2|x|+1}dx=2\int_0^1\frac{x+1}{x^2+2x+1}dx=2\int_0^1\frac{x+1}{(x+1)^2}dx=2\int_0^1\frac{1}{x+1}dx$ [1]

$\Rightarrow I=2\left[\log|x+1|\right]_0^1=2\left[\log 2-\log 1\right]$ $\therefore I=2\log 2$. $(\because \log 1=0)$ [1/2]

Q21. Here $\cos y\,dx+(1+2e^{-x})\sin y\,dy=0$ $\Rightarrow\int\frac{dx}{1+2e^{-x}}=-\int\frac{\sin y}{\cos y}dy$ $\Rightarrow\int\frac{e^x dx}{e^x+2}=-\int\frac{\sin y}{\cos y}dy$ [1]

In first integral, put $e^x+2=t\Rightarrow e^x dx=dt$

Also, in second integral, put $\cos y=u\Rightarrow-\sin y\,dy=du$

$\therefore\int\frac{dt}{t}=\int\frac{du}{u}$ $\Rightarrow\log|t|=\log|u|+\log C$

$\Rightarrow\log\left|e^x+2\right|=\log|C\cos y|$ $\Rightarrow e^x+2=C\cos y$ [1½]

As $y(0)=\frac{\pi}{4}$, so $e^0+2=C\cos\frac{\pi}{4}$ $\Rightarrow C=3\sqrt{2}$ [1]

Hence the required particular solution is : $e^x+2=3\sqrt{2}\cos y$. [1/2]

OR $\frac{dx}{dy}=\frac{y\tan y-x\tan y-xy}{y\tan y}$ $\Rightarrow\frac{dx}{dy}=\frac{y\tan y-x(\tan y+y)}{y\tan y}=1-x\left(\frac{1}{y}+\frac{1}{\tan y}\right)$

$\Rightarrow\frac{dx}{dy}+x\left(\frac{1}{y}+\frac{1}{\tan y}\right)=1$ $\therefore P(y)=\frac{1}{y}+\frac{1}{\tan y}$, $Q(y)=1$ $[\because\frac{dx}{dy}+P(y)x=Q(y)$ [1]

Now I.F. $=e^{\int\left(\frac{1}{y}+\cot y\right)dy}=e^{\log y+\log\sin y}=e^{\log y\sin y}=y\sin y$ [1]

So, the solution is : $x(y\sin y)=\int 1\times y\sin y\,dy+C$

$\Rightarrow xy\sin y=y\int\sin y\,dy-\int\left(\frac{d}{dy}(y)\int\sin y\,dy\right)dy+C$ [1/2]

$\Rightarrow xy\sin y=-y\cos y+\int\cos y\,dy+C$ $\Rightarrow xy\sin y=\sin y-y\cos y+C$ [1½]

That is, $x = \dfrac{\sin y - y\cos y + C}{y\sin y}$.

Q22. Let $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$.

As $\vec{r} \perp \vec{q}$ so, $\vec{r}.\vec{q} = 0$ i.e., $a - 2b + c = 0$...(i) [1]

Also $\vec{r}$ is coplanar with vectors $\vec{p}$ and $\vec{q}$ so, $[\vec{p}\ \vec{q}\ \vec{r}] = 0$

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ a & b & c \end{vmatrix} = 0 \quad \Rightarrow 3a - 3c = 0$ i.e., $a = c$...(ii) [1]

By (i) and (ii), we get : $b = c$

$\therefore$ the direction ratios of $\vec{r}$ are a, b, c i.e., c, c, c i.e., $1,1,1$

So, $\vec{r} = \hat{i} + \hat{j} + \hat{k}$ [1]

Now, the required vector has magnitude of $5\sqrt{3}$ so, required vector is $5\sqrt{3} \times \hat{r}$

Therefore, required vector $= 5\sqrt{3} \times \dfrac{\vec{r}}{|\vec{r}|} = 5\sqrt{3} \times \dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = 5\hat{i} + 5\hat{j} + 5\hat{k}$. [1]

Q23. $\because$ Vector eq. of the line joining two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

So, $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\{(-3\hat{i} + 4\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})\}$ i.e., $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-4\hat{i} + 2\hat{j})$ ...(i) [2]

Now the d.r.'s of line (i) are –4, 2, 0. Also the d.r.'s of z-axis are 0, 0, 1. [1]

Since $(-4)(0) + (2)(0) + (0)(1) = 0$ so, clearly line (i) is perpendicular to the z-axis. [1]

(By using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, condition for perpendicular lines).

SECTION D

Q24. For the given matrix A, $|A| = \begin{vmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{vmatrix} = 3(-2) - 1(3) + 2(-4) = -17 \neq 0 \quad \therefore A^{-1}$ exists. [1½]

Consider A_{ij} be the cofactor of the element a_{ij} of matrix A.

$A_{11} = -2,\ A_{12} = -3,\ A_{13} = -4,$
$A_{21} = 1,\ A_{22} = -7,\ A_{23} = 2$
$A_{31} = -7,\ A_{32} = 15,\ A_{33} = 3$

$\therefore \text{adj.}A = \begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix}$ [2]

So, $A^{-1} = \dfrac{\text{adj.}A}{|A|} = \dfrac{1}{-17}\begin{bmatrix} -2 & 1 & -7 \\ -3 & -7 & 15 \\ -4 & 2 & 3 \end{bmatrix} = \dfrac{1}{17}\begin{bmatrix} 2 & -1 & 7 \\ 3 & 7 & -15 \\ 4 & -2 & -3 \end{bmatrix}$ [1/2]

Now consider the equations : $3x + 3y + 2z = 1,\ x + 2y = 4,\ 2x - 3y - z = 5$

Let $P = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & -1 \end{bmatrix} = A^T$, $B = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$ and, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Since $PX = B \Rightarrow X = P^{-1}B = (A^{-1})^T B$ $\quad [\because P^{-1} = (A^T)^{-1} = (A^{-1})^T$ [1/2]

So, $X = \dfrac{1}{17}\begin{bmatrix} 2 & 3 & 4 \\ -1 & 7 & -2 \\ 7 & -15 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \Rightarrow X = \dfrac{1}{17}\begin{bmatrix} 34 \\ 17 \\ -68 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$ [1]

By equality of matrices, we get : $x = 2,\ y = 1,\ z = -4$. [1/2]

OR Let $A=\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$

As $A=IA$ so, $\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$ [1/2]

By $R_1 \to R_1+R_3$, $\begin{bmatrix}-1 & 1 & 6\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$ [1/2]

By $R_1 \to (-1)R_1$, $\begin{bmatrix}1 & -1 & -6\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}=\begin{bmatrix}-1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$ [1/2]

By $R_2 \to R_2+5R_1$, $R_3 \to R_3+3R_1$, $\begin{bmatrix}1 & -1 & -6\\ 0 & -2 & -29\\ 0 & -1 & -15\end{bmatrix}=\begin{bmatrix}-1 & 0 & -1\\ -5 & 1 & -5\\ -3 & 0 & -2\end{bmatrix}A$ [1]

By $R_2 \leftrightarrow R_3$, $\begin{bmatrix}1 & -1 & -6\\ 0 & -1 & -15\\ 0 & -2 & -29\end{bmatrix}=\begin{bmatrix}-1 & 0 & -1\\ -3 & 0 & -2\\ -5 & 1 & -5\end{bmatrix}A$ [1/2]

By $R_2 \to (-1)R_2$, $\begin{bmatrix}1 & -1 & -6\\ 0 & 1 & 15\\ 0 & -2 & -29\end{bmatrix}=\begin{bmatrix}-1 & 0 & -1\\ 3 & 0 & 2\\ -5 & 1 & -5\end{bmatrix}A$ [1/2]

By $R_1 \to R_1+R_2$, $R_3 \to R_3+2R_2$, $\begin{bmatrix}1 & 0 & 9\\ 0 & 1 & 15\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}2 & 0 & 1\\ 3 & 0 & 2\\ 1 & 1 & -1\end{bmatrix}A$ [1]

By $R_1 \to R_1-9R_3$, $R_2 \to R_2-15R_3$, $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}A$ [1]

Since $I=A^{-1}A$ $\therefore A^{-1}=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}$. [1/2]

Q25. Let the length and breadth of the base be x and the height of the godown be y.
Let C be the cost of constructing the godown and V be the given volume. [1/2]

So, $V=x^2y$ $\Rightarrow xy=\dfrac{V}{x}$...(i) [1/2]

And, $C=(3x^2+4xy)\times k$, where k is cost incurred for walls.

$\therefore C=\left(3x^2+\dfrac{4V}{x}\right)\times k$ [By (i)] [1]

Now $\dfrac{dC}{dx}=\left(6x-\dfrac{4V}{x^2}\right)\times k$ and, $\dfrac{d^2C}{dx^2}=\left(6+\dfrac{8V}{x^3}\right)\times k$ [1 + 1]

For maximum and/or minimum value of C, $\dfrac{dC}{dx}=0$ $\Rightarrow 6x-\dfrac{4V}{x^2}=0$

$\Rightarrow x^3 = \frac{2V}{3} \quad \therefore x = \left(\frac{2V}{3}\right)^{1/3}$ [1]

When $x^3 = \frac{2V}{3}$, then $\frac{d^2C}{dx^2} = \left(6 + 8 \times \frac{3}{2}\right) \times k = 18k > 0$ ($\because$ k is cost, so $k > 0$) [1/2]

So, C is minimum when $x = \left(\frac{2V}{3}\right)^{1/3}$.

Now, $V = x^2 y \Rightarrow \frac{3x^3}{2} = x^2 y \Rightarrow x^2\left(\frac{3x}{2} - y\right) = 0 \Rightarrow \frac{3x}{2} = y \therefore y = \frac{3}{2}\left(\frac{2V}{3}\right)^{1/3}$ [1/2]

Hence the dimensions of the godown are $\left(\frac{2V}{3}\right)^{1/3} \times \left(\frac{2V}{3}\right)^{1/3} \times \frac{3}{2}\left(\frac{2V}{3}\right)^{1/3}$.

Q26. We have $y = \sqrt{x}$...(i), $2y + 3 = x$...(ii)

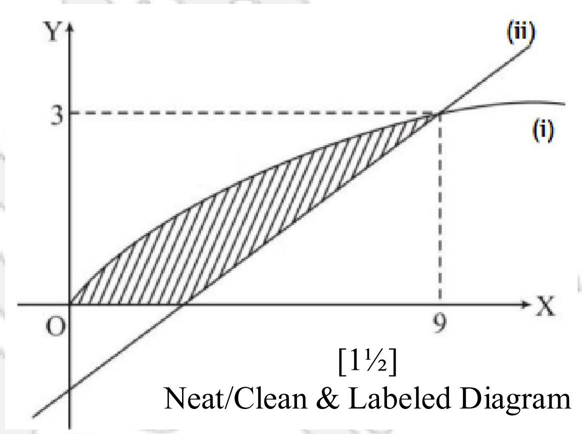
Solving (i) and (ii), $2\sqrt{x} + 3 = x \Rightarrow (x-3)^2 = 4x$

$\Rightarrow x^2 - 10x + 9 = 0 \Rightarrow (x-9)(x-1) = 0$

$\therefore x = 1, 9 \Rightarrow y = -1, 3$

But note that $y = \sqrt{x}$ so, $y > 0$

Therefore, the point of intersection is (9, 3). [1]



[1½]
Neat/Clean & Labeled Diagram

Now, required area $= \int_0^3 (2y+3)dy - \int_0^3 y^2 dy$ [1½]

$\Rightarrow \quad = \left[\frac{(2y+3)^2}{2 \times 2} - \frac{y^3}{3}\right]_0^3$ [1]

$\Rightarrow \quad = \left[\frac{81}{2 \times 2} - \frac{27}{3}\right] - \left[\frac{9}{2 \times 2} - 0\right] = \frac{72}{2 \times 2} - \frac{27}{3} = 18 - 9$ [1]

$\Rightarrow \quad = 9$ Sq. units.

OR We have $\{(x, y) : x^2 + y^2 \le 8, x^2 \le 2y\}$.

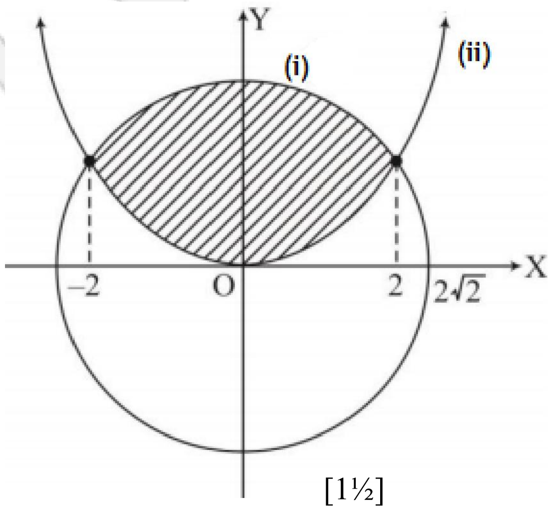
Consider $x^2 + y^2 = 8$...(i), $x^2 = 2y$...(ii)

Solving (i) and (ii), $2y + y^2 = 8 \Rightarrow y^2 + 2y - 8 = 0$

$\Rightarrow (y+4)(y-2) = 0 \quad \therefore y = 2, -4$

$\because y = \frac{x^2}{2} > 0$ so, $y = 2 \Rightarrow x = \pm 2$

So, points of intersections are (2, 2), (–2, 2). [1]



[1½]
Neat/Clean & Labeled Diagram

Now, required area $= \int_{-2}^{2} \sqrt{8 - x^2}\,dx - \int_{-2}^{2} \frac{x^2}{2} dx$ [1]

$\Rightarrow \quad = 2\left\{\int_0^2 \sqrt{8 - x^2}\,dx - \int_0^2 \frac{x^2}{2} dx\right\}$

(Note that, in both the integrals, the functions are even functions).

$\therefore$ Req. area $= 2\left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}} - \frac{x^3}{6}\right]_0^2$ [1½]

$\Rightarrow \quad = 2\left\{\left[2 + 4 \times \frac{\pi}{4} - \frac{8}{6}\right] - [0 + 0 - 0]\right\} = \left\{2\pi + \frac{4}{3}\right\}$ Sq. units. [1]

Q27. Let $\frac{x-1}{3}=\frac{y-4}{2}=\frac{z-4}{-2}$ …(i) and $\frac{x+1}{2}=\frac{1-y}{4}=\frac{z+2}{1}$ …(ii)

The d.r.'s of lines (i) and (ii) are respectively, 3, 2, –2; 2, –4, 1.

Let A, B, C be the d.r.'s of the Normal to the plane containing line (i). So, the required plane will surely contain the point (1, 4, 4), which is a point on line (i).

$\therefore \pi : A(x-1)+B(y-4)+C(z-4)=0$...(iii) [1]

Since the line (i) is on the plane (iii) so, the normal to the plane will also be perpendicular to the line (i).

$\therefore 3A+2B-2C=0$...(iv) [1/2]

Also, line (ii) is parallel to the plane (iii) so, Normal to the plane will be perpendicular to the line (ii).

$\therefore 2A-4B+1C=0$...(v) [1/2]

By 2 × (iv) + (v), we get : $A=\frac{3C}{8}$. Also $4B=2A+C=\frac{3C}{4}+C=\frac{7C}{4}$ i.e., $B=\frac{7C}{16}$.

So, the d.r.'s of normal are A, B, C i.e., $\frac{3C}{8}, \frac{7C}{16}, C$ i.e., 6, 7, 16. [1]

By (iii), the req. plane is $\pi : 6(x-1)+7(y-4)+16(z-4)=0$ i.e., $6x+7y+16z-98=0$. [1]

Note that on the line (ii), a point is (–1, 1, –2).

Since the line (i) lies on the plane (iii) and (ii) is parallel to this plane so, the shortest distance between lines (i) and (ii) will be **same** as the distance of point (–1, 1, –2) from the plane.

Hence, S. D. $=\frac{|6(-1)+7(1)+16(-2)-98|}{\sqrt{6^2+7^2+16^2}}=\frac{|-6+7-32-98|}{\sqrt{36+49+256}}=\frac{129}{\sqrt{341}}$ units. [2]

OR Let $\pi_1 : x+2y+3z=8$ and $\pi_1 : 2x+3y+4z=11$.

Let's first of all, find a point which satisfies both these planes. This point will lie on the **line** of intersection of these two planes.

Consider $x+2y=8-3z$...(i), $2x+3y=11-4z$...(ii)

By $2\times$(i) – (ii), we get : $y=5-2z \;\; \therefore x=z-2$

That is the required point is (x, y, z) i.e., $(z-2,\ 5-2z,\ z) \quad \therefore (-2, 5, 0)$

(**Note** that, we have put z = 0. You can put any value of z in order to obtain the desired point).

Let P(–2, 5, 0). As this point satisfies both the planes so, it must lie on the **line** of intersection of these planes.

Also the d.r.'s of normals to the planes (i) and (ii) are respectively 1, 2, 3; 2, 3, 4.

So, $\vec{m}_1=\hat{i}+2\hat{j}+3\hat{k},\ \vec{m}_2=2\hat{i}+3\hat{j}+4\hat{k} \qquad \therefore \vec{m}_1\times\vec{m}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 2 & 3 & 4\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$.

That is, the d.r.'s of the **line** of intersection of planes will be –1, 2, –1 i.e., 1, –2, 1.

Hence the eq. of **line** of intersection of the planes (i) and (ii) is : $\frac{x+2}{1}=\frac{y-5}{-2}=\frac{z}{1}$ …(iii)

Let $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ …(iv)

If line (iii) and (iv) are coplanar then, we have $\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$.

LHS : $\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=\begin{vmatrix}-2-(-1) & 5-(-1) & 0-(-1)\\ 1 & 2 & 3\\ 1 & -2 & 1\end{vmatrix}=\begin{vmatrix}-1 & 6 & 1\\ 1 & 2 & 3\\ 1 & -2 & 1\end{vmatrix}$

$\Rightarrow \quad = -1\times 8 - 6\times(-2) + 1\times(-4) = 0 = \text{RHS}$.

So, the **line** of intersection of planes (i) and (ii) (i.e., the line (iii)) is coplanar with the line (iv).
Now for the equation of plane containing lines (iii) and (iv), let's find the normal vector to the required plane.

For line (iii) and (iv), we have : $\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$.

So, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 8\hat{i} + 2\hat{j} - 4\hat{k} = \vec{m}$ (say, Normal vector).

Also a point on the required plane may be taken from any one of the lines (iii) or, (iv). Let's take a point from line (iii). Clearly (–2, 5, 0) i.e., $\vec{a} = -2\hat{i} + 5\hat{j}$ will lie on the required plane too.

Now required eq. of plane : $\vec{r}.\vec{m} = \vec{a}.\vec{m} \quad \Rightarrow \vec{r}.(8\hat{i} + 2\hat{j} - 4\hat{k}) = (-2\hat{i} + 5\hat{j}).(8\hat{i} + 2\hat{j} - 4\hat{k})$

$\Rightarrow \vec{r}.(4\hat{i} + \hat{j} - 2\hat{k}) = -3$ i.e., $4x + y - 2z + 3 = 0$.

Alterantive : In order to show that the given line $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ ...(i) is coplanar with the **line** determined by the intersection of the planes $\pi_1 : x + 2y + 3z = 8$ and $\pi_1 : 2x + 3y + 4z = 11$, we'll show that there exists a plane which contains the line (i) and passes through the intersection of planes π_1 and π_2.

Eq. of plane through the planes π_1 and π_2 is, $\pi : x + 2y + 3z - 8 + \lambda(2x + 3y + 4z - 11) = 0$...(ii) [2]

If line (i) is on the plane (ii) then, the point on this line (–1, –1, –1) must satisfy the plane (ii).
That is, $-1 - 2 - 3 - 8 + \lambda(-2 - 3 - 4 - 11) = 0 \Rightarrow \lambda = -7/10$

By (ii), we get : $\pi : 4x + y - 2z + 3 = 0$...(iii) [2]

Now d.r.'s of the line (i) are 1, 2, 3 and the d.r.'s of normal to the plane (iii) are 4, 1, –2.

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get : $1\times 4 + 2\times 1 + 3\times(-2) = 0$. [1]

It implies that the line (i) lies in the plane (iii), as the normal to the plane is perpendicular to the line too.

$\therefore$ the two lines are coplanar and the equation of plane containing them is $4x + y - 2z + 3 = 0$. [1]

Q28. Let the manufacturer makes x and y number of toy A and toy B, respectively.

To maximize : Z = ₹ (50 x + 60 y) [1]

Subject to constraints : $x, y \geq 0$;

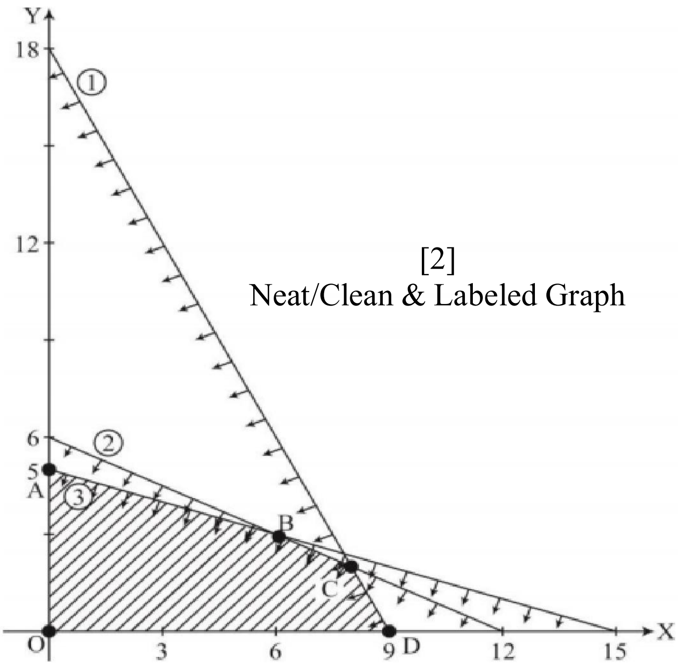
$20x + 10y \leq 180$...(1),

$10x + 20y \leq 120$...(2), [1½]

$10x + 30y \leq 150$...(3)

Corner Points	Value Of Z (in ₹)
O(0, 0)	0
A(0, 5)	300
B(6, 3)	480
C(8, 2)	520 ← Max.
D(9, 0)	450

[1]



[2]
Neat/Clean & Labeled Graph

Hence, the maximum profit is ₹ 520.

And, nos. of toy A and B are respectively, 8 and, 2. [1/2]

Q29. Let E : the car needs service and tuning. Let E_1, E_2, E_3 be the events that car is rented from agency X, Y, Z respectively. [1/2]

So, $P(E_1)=50\%,\ P(E_2)=30\%,\ P(E_3)=20\%,\ P(E|E_1)=9\%,\ P(E|E_2)=12\%,\ P(E|E_3)=10\%$. [2½]

By Bayes' Theorem, $P(E_3|E)=\dfrac{P(E_3)P(E|E_3)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)+P(E_3)P(E|E_3)}$ [1]

$$\Rightarrow P(E_3|E)=\frac{\frac{20}{10}\times\frac{10}{100}}{\frac{50}{10}\times\frac{9}{100}+\frac{30}{10}\times\frac{12}{100}+\frac{20}{10}\times\frac{10}{100}}=\frac{20}{45+36+20}=\frac{20}{101}$$ [1]

$$\therefore P(\overline{E}_3|E)=1-P(E_3|E)=1-\frac{20}{101}=\frac{81}{101}.$$ [1]

Note that $\overline{E}_3$ is the event that car is **not** rented from agency Z.

■

This Sample Paper has been issued by **CBSE, New Delhi** for **2019 Board Exams of XII**.
Note : We've **re-typed** the same and have **added more illustrations** in the solutions.
On other hand, if you find any error which could have gone un-noticed, please do inform us via **WhatsApp @ +919650350480** or **Email us : iMathematicia@gmail.com**

For video lectures, please visit YouTube.com/@theopgupta